MATHEMATICS TEACHING

THE BULLETIN OF THE
ASSOCIATION for TEACHING AIDS in MATHEMATICS

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EDITORIAL

THE SUPPLY OF TEACHERS OF MATHEMATICS

A civilisation such as ours, dependent on scientific technology for its survival and on mathematical techniques of many kinds for its administration and government, requires an educational system which can produce an élite corps of mathematicians, scientists and technologists (all of whom demand a sound mathematical education) and which can develop such knowledge and understanding on the part of the ordinary citizen as will enable him to appreciate the needs of his society, to use intelligently the devices of its technologists and critically to appraise the advice of its experts. Such a system must have regard to its own needs by absorbing as teachers those students who by their attitudes and attainments will enable it to achieve its ends.

Some estimate of the quality of the teachers we shall have may be determined from a survey of students training to become teachers. Such an estimate has been presented in the recent report* published by the Association of Teachers in Colleges and Departments of Education, and gloomy reading it is to those who are concerned about the teaching of mathematics.

Take first the primary school teachers, those who lay the foundations of mathematics and communicate a love of the subject and a confidence that will inspire their pupils. "The utter insensitivity of so many students in the realm of mathematics presents a most difficult problem and their lack of success with enlightened methods of teaching tends to bring these methods into disrepute. This statement of the problem implies that modern methods of arithmetic teaching make demands upon teachers which some students seem never likely to be able to meet. They may achieve some limited success with simple number work but their teaching will inevitably be barren of mathematical ideas and modes of thought." (p. 39.) Three-quarters of the girls who leave the grammar schools for training colleges will eventually staff the primary schools. Half of them (approximately) have taken an examination in mathematics at G.C.E. but the majority have an active dislike of the subject. Women students "fear", "are frightened of", "shun", "dislike", "prefer almost anything to" mathematics and a high proportion of them show "gross incompetence outside a narrow range of stock examination question routine."

Secondary Modern Schools look to students who have made some special study of mathematics. For boys' schools the position is rather better than for girls'; 14.1 per cent of the male intake who make a special study of mathematics is sufficient to maintain the present position but the 3.8 per cent of the women's intake means that two girls in three in the secondary modern schools are unlikely to come into contact with anyone who has studied mathematics. Already many girls' schools are relying on teachers who teach from conscription rather than from inclination or because of a liking of the subject.

The grammar schools are in no better shape. The supply of mathematics teachers might appear to be adequate but the quality is such that not more than

* *The Supply of Mathematics and Science Teachers*—Association of Teachers in Colleges and Departments of Education (Methuen, 1956).

one boy in four is likely to enjoy the stimulus of a man with a 'good' honours degree. It is significant that on the average about forty times as many well qualified men apply for senior posts on the arts side as for those on the maths. and science sides. With this state of affairs it will not be surprising if more boys elect to specialise in the arts. Already in the U.S.A., where from similar staff shortages, 80 per cent of high schools teach no mathematics beyond ordinary arithmetic, the numbers of students presenting themselves for courses in science and engineering are beginning to fall.

In the girls' grammar schools, the situation is exceedingly serious. The total number of maths. teachers is apparently dwindling at the rate of fifty a year. This deficiency, if it is to be made up, will absorb many of the mathematics students from the two-year colleges.

The training colleges, themselves, suffer a grave shortage of mathematics specialists. There is thus developing a vicious circle: a circle that is difficult to break, the supply of qualified recruits depends on the supply of qualified teachers, which in turn depends on the supply of qualified recruits. What is to be the solution? Peripatetic ..aths. teachers? Half-time teachers as in Staffordshire? Direction of students? Higher pay? The report suggests that the Training Colleges could make a start by increasing the weight they give to mathematics, by forming committees to attack the problem at all levels and by endorsing the unsuitability of otherwise well qualified teachers as teachers of mathematics.

Courses for teachers in service would form a valuable contribution. It is in this field that we as individuals and as an association can help. Ultimately we as teachers must accept the burden of our own responsibility in whatever educational establishment we are employed, by assisting new entrants to the profession, supporting weekend courses and conferences, by attending lectures as they are organised and by constructively and openly criticising the mathematical education we see around us.

C.H.

Mathematics is the majestic structure conceived by man to grant him comprehension of the universe. It holds both the absolute and the infinite, the understandable and the forever elusive. It has walls before which one may pace up and down without result; sometimes there is a door: one open it—enters—one is in another realm, the realm of the gods, the room which holds the key to the great systems. These doors are the doors of the miracles. Having gone through one, man is no longer the operative force, but rather it is his contact with the universe. In front of him unfolds and spreads out the fabulous fabric of numbers without end. He is in the country of numbers. He may be a modest man and yet have entered just the same. Let him remain, entranced by so much dazzling, all pervading light.

More than these thirty years past, the sap of mathematics has flown through the veins of my work, both as an architect and painter; for music is always present within me. (Let me explain here that at school I was very bad at mathematics; the subject only filled me with misery and distaste.)

LE CORBUSIER, *The Modulor.*

MATHEMATICS IN THE SECONDARY SCHOOL CURRICULUM

I—WHITHER MATHEMATICS TEACHING?

R. H. COLLINS

"From the time of Plato to the present day the place of mathematics in a liberal education has never been seriously disputed. The teacher of Mathematics has not been obliged, like his classical colleague to defend his subject from external attack; nor has he, like the teacher of Geography for instance, had a struggle to rescue it from undeserved neglect. The well known existence of numberless applications of Mathematics to commercial and industrial affairs, increasing and largely contributing to the progress of civilization, carries sufficient conviction of the importance of the subject to the man of the world; while the dictum of Bacon that Mathematics 'make men subtle' sums up a widely accepted view that satisfies those who are more concerned for the mental and moral than for the material results of education. It is then little matter for surprise that the movement for the reform of mathematical teaching . . . should have been mainly directed from within. The teacher gifted with some power of self-criticism could not rest satisfied with the easy acceptance of his practice by an indulgent and unenlightened public opinion. He was bound to ask himself whether there was any evidence amongst the pupils that passed through his hands of that subtlety which the study of Mathematics was alleged to produce, and on a fair review of his experience as a teacher he was compelled to admit that, whereas stupidity and fatheadedness as the outcome of his ministrations were of constant occurrence, the production of subtlety were phenomenally rare. Mathematics as a cure for a 'wandering wit' might be all very well for a man of the intellectual capacity of Bacon, but somehow it seemed to fail in its effect when applied to the ordinary schoolboy or girl. Nor when he looked at his subject from the utilitarian standpoint was he any happier in an attempt to justify his existence. He could not honestly say that to 90 per cent of his pupils, apart from arithmetic, the stuff he was teaching was ever likely to be of material service—an impression which conference with mathematicians engaged in the practical application of the subject only served to intensify.

"Whether or not this was unduly pessimistic, it was abundantly clear as the results produced were in no way commensurate with the labour involved in producing them."

It is hard to believe that this passage is taken from a publication* which appeared some fifty years ago for it would seem that many of the same questions mentioned therein are still being as forcibly asked by today's teachers of Mathematics. The modern answers however are likely to be vastly different from the replies of yesteryear. Indeed a civilization is decadent if ever its educational system loses sight of the fact that it has a prime responsibility to note changes in its cultural setting and has to prepare the younger generation for the part they will have to play in the even newer situation in which they will find themselves a decade away.

But before any answers can be given to the question "What and how shall we teach in mathematics?" it is essential to become quite clear as to what are the motives for teaching the subject at all. In so doing the enquirer must be constantly on his guard not to take shelter behind the arguments used by his illustrious forbears lest he fail in his duty towards the present generation. To do this is equally as dangerous as the practice of making suggestions for items to be included in the School syllabus without the slightest attempt to ensure that the teachers involved are satisfied with the grounds on which these suggestions are made. This is far too often a fault of those in authority and other high places in the profession and it is hoped in succeeding issues of *Mathematics Teaching* to remedy this defect by

* *New Teaching*, edited by Sir John Adams. Chapter on *Mathematics* by G. James Strachan.

affording an opportunity for some possible teaching motives to be made known.

Before going much farther it ought to be made quite clear as to what is being meant by the two terms "Mathematics" and "Secondary School."

MATHEMATICS. It is intended that the term shall be as wide as possible and cover not only arithmetic but also algebra, geometry, trigonometry, co-ordinate geometry, calculus, statistics, topology, etc., since it seems that there is an increasing pressure that parts of these subjects should find their way into every curriculum. This definition needs to be carefully borne in mind when considering some of the aims given in former times since this or that particular branch of mathematics may not have reached the necessary stage of development to enable a systematic approach to be worked out so that it could be included in the educational curriculum of that previous generation.

SECONDARY EDUCATION. It would be easy to limit the meaning of the words so as to deal solely with what was formerly called "Grammar School curriculum" with its examination goal. But in these circumstances the aim is easily formulated, namely the taking of a specified examination, and the content of the syllabus and thereafter the methods of teaching are then automatically determined. Today in the Secondary Grammar School the abandonment of the old School Certificate Examination and its replacement by the higher standard of the General Certificate of Education has meant that many children are leaving such schools without having taken the examination or have failed to reach the necessary standard for a pass. In consequence the specialist teachers in these schools are being forced to consider their motives more critically. Indeed with this mainstay of educational incentive eliminated here, it is likely that in more and more sections of their work the aims now formulated by these specialists may find some identity of purpose with their non-specialist mathematical friends in the Secondary Modern School. The children involved will form some 85-90 per cent of the future population of this country. In this second field, too, the teachers have been made more concious of a teaching problem by the raising of the school leaving age from 14 to 15 combined with a more enlightened conception of what is meant by "necessary arithmetic." Another feature of the situation is a general desire to break away from the old authoritarian methods of pedagogy (Grecian in their outlook) in order to make use of the newer approach founded on the philosophies of the 18th century and subsequently crystallized into the school situation by Pestalozzi and others. In this study, therefore, an endeavour is being made to look for those aims which would justify the teaching of the subject to *all* children; i.e. to look for a general educational value as distinct from a vocational one. For unless it can be shown that the study of mathematics beyond the ability to count, add, subtract, divide, and multiply simple whole numbers and easy fractions, and ability to perform simple money, weight and measurement calculations is of value as a means of providing a contribution to a liberal education, that is of value to every man, woman and child whatever the age, occupation, position on the social scale or mental attainment, then the claim of Mathematics to continue to occupy a large proportion of the timetable in every type of school cannot be upheld. For then the study of the more advanced portions of the subject becomes either useless or mainly vocational, in which respect it is

bound to vary from one child to the next. Since it is impossible to give every child his own particular piece of vocational mathematics, which will vary from zero up to the full requirements of the future mathematician, then much of the later work covered in the normal secondary school is wasted for the vast majority of the pupils. The syllabus then covered in the school is without valid purpose save that of the necessity of going through the meaningless school curriculum, a process which brings in its train lack of understanding, frustration and consequent ill-being for the best development of the child.

In order to look into the suggested aims for teaching the subject the following procedure will be adopted for the subsequent portions of this work. A list of the main reasons given by various people in the past has been drawn up, compiled from a variety of sources, some renowned, others less so, some recent, others extending back to Plato. It was not by any means a comprehensive survey of all literature on the subject since it is quite impossible to follow up all the many references found without writing a complete book on the subject. Indeed, little or no account was taken of the attitude held by the many American writers on the subject since it was discovered that they were taking up a stand vastly different from writers in the Old World. However, judging from the quotation at the beginning of this article it is unlikely that as far as the British educational system is concerned much that is important will have been omitted. Indeed, it was noted that the same reasons, with almost similar arguments were developed time and time again. It is therefore unlikely that any important considerations have been overlooked.

Having listed each of these aims some attempt will be made to consider the assumptions (if any) upon which they are based and to see how far these are valid at the present day. At the same time we should try to see to what extent the achievement of these objectives would depend on (a) content of the syllabus, (b) the exact method of presentation of the material, and to see if these two important factors did, or are now being given their proper attention. For it is quite apparent that the inclusion of a specific subject in the curriculum with certain well defined aims, may completely fail to achieve these purposes through bad teaching or poor content on which the teacher is expected to function.

It will also be necessary to take some account of the form of educational system then existing and the social setting at the period when the reasons were advanced. If there have been changes in either of these factors since then an attempt will be made to find out in what respect the arguments given are still valid or how they should be modified.

In trying to decide on the list of aims to be discussed it was often found difficult to get a heading which would be in complete agreement with the exact wording and interpretation of all the sources investigated. It must therefore be realised that a judgment has had to be made as to the exact headings to use and that it was thought fit to ignore altogether some headings which were considered to be of minor importance.

This then is the extent of the problem to be discussed and this introduction is brought to a conclusion with a request that all teachers, particularly from Secondary Modern Schools, should contribute their comments on this series and their views

on the reasons for teaching the subject. Such articles will indeed be welcome since it is only from the concerted efforts of all members of the profession, combining experience in a variety of schools, that an adequate assessment of the subject aims can be made. Moreover, it should be emphasised that these articles are bound to be based on the personal opinions of the writer who may not necessarily be correct in all his judgments. The more that is written and printed the more likely it will be that lasting benefit will accrue to all teachers of the subject.

(To be continued)

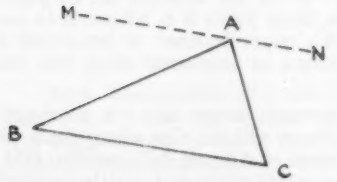
GEOMETRICAL ANAGRAMS

IAN HARRIS

To help familiarise my pupils with the formal setting out of a Geometrical argument I have found the following type of problem useful.

In this Geometrical puzzle take the first letter of each missing word or group of letters, re-arrange, and make a word which has something to do with the problem.

THE ANGLE SUM OF A ---- ---- ---- ---- ANGLES



GIVEN ΔABC

TO PROVE $\angle ABC + \angle BCA + \angle CAB = 180°$

CONSTRUCTION

Draw MAN parallel to BC.

PROOF $\angle ABC = \angle MAB$ (---- angles, MN parallel to BC)

also $\angle BCA =$ --- (---- angles, MN parallel to BC)

and $\angle CAB = \angle CAB$ (equal angles).

Adding $\angle ABC + \angle BCA + \angle CAB = \angle MAB + \angle CAB + \angle NAC$

but $\angle MAB + \angle CAB + \angle NAC = 180°$ (straight ---- angle).

$\therefore \underline{\angle ABC + \angle BCA + \angle CAB = 180°.}$

Note : For the ---- triangle (all sides and angles equal) each angle is 60°.

SOLUTION :

*T*riangle, *E*quals, *T*wo, *R*ight, *A*lternate, $\angle$*N*AC, *A*lternate, *L*ine, *E*quilateral.

ALTERNATE.

MATERIALS FOR MODELS

J. W. PESKETT

The words "Mathematical Model" can be used in a very limited sense but in this series of articles will include any object whatsoever that can be used as an aid in the teaching of mathematics. The object need not be a complicated one, in fact the simpler the better. An ordinary sheet of foolscap can be used to demonstrate a rectangle, or by suitable rolling a cylinder or cone. The question of durability (that is, should the model be made in paper or metal) or the question of artistic appeal (that is, should the model be painted and polished or left plain) will be for the model-maker to decide. The model-maker may be the pupil or the teacher; in either case, each will be a "learner"! The main purpose of these articles, however, is to talk about various materials and methods of using them.

Perhaps one of the most versatile of materials and the easiest to handle is cardboard, ranging from the thickness of paper to card nearly one quarter of an inch thick. It may be obtained from the covers of old exercise books, manilla folders, old boxes or a better quality bought from a stationers. It is surprising too, how rigid the models can be, if properly constructed and reinforced, even using thin card. The tools required are a sharp knife (there are several kinds on the market with interchangeable blades of razor-sharpness, but old razor blades are not advised, being too pliable), a steel rule as a guide when cutting straight lines, perhaps a leather punch to make neat holes, and, of course, drawing instruments. For assembling the various parts together there are many adhesives available but perhaps the best is balsa cement, as used in making model aeroplanes, it dries very quickly and gives a strong joint. In some cases it is possible to use the adhesive brown paper strip, Sellotape, or even a stapling machine. For hinge joints it is advisable to use a leather punch to make the holes and to insert a paper fastener, or better still a small metal eyelet, as can be bought at a stationers or handicraft shop, but this latter needs a special tool to fasten it.

Here are two easy models for use in a geometry lesson. Cut out a triangle ABC of convenient size in cardboard, any thickness will do. Cut along the lines PQ, PN, and QM, where P and Q are the mid points of AB and AC, and PN, QM, are perpendicular to BC. (Fig. 1.) Replace the portions again to form the original

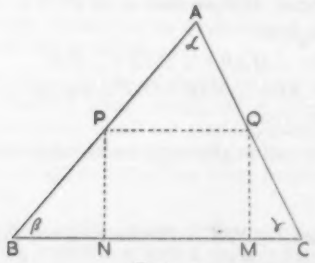


Fig. 1.

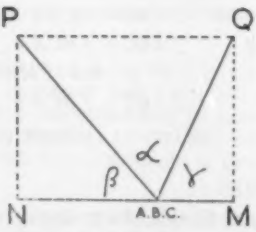


Fig. 2.

triangle and carefully put Sellotape along the cuts. The vertices of the triangle can now fold along the cuts to meet on the line BC and demonstrate that the three angles of a triangle add up to 180 degrees. (Fig. 2.)

For the next model, cut four equal strips say about eight inches by half an inch. Punch holes near the end of each strip and use paper fasteners to form the four strips into a rhombus. If "shirring" elastic is now used for the diagonals the variable rhombus can be used to demonstrate that the diagonals are always at right angles. (Fig. 3.)

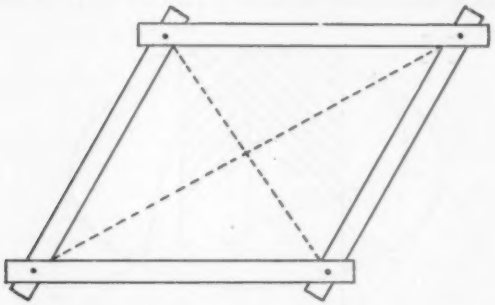


Fig.3

Both these models, of course, would last longer if made of stronger materials, but many a model can be made in cardboard at first to see how it works, and then if thought necessary, made in a more substantial form. If children are doing the construction (when time is available it will prove of great value), then cardboard is easier for them to work with, or they may be challenged to make a "better" model in their spare time.

It is not necessary to reproduce here the networks for the five platonic solids, as they can be found in many books on geometry. A point or two about their construction, however, might be worth considering. For the greatest accuracy in the shape of the solid it is best to cut each face separately, especially if thick card is being used. The edges are then joined on the outside or the inside by means of Sellotape or gummed paper strip. If they are joined on the inside (which usually makes a neater model) the last face can be put in position by sticking flaps inside the last opening and attaching the last face by balsa cement. If thin card is used (and it is surprising how rigid the final model can be, even with very thin card) then certain faces can be left joined together when cutting out the network and flaps left attached for joining other faces. It is best to "score" (i.e., cut very nearly through the card) the edges along which the card is going to be bent. Of the five

platonic solids the dodecahedron is the hardest to make, and great accuracy is required in the preliminary drawing. After experience has been acquired with some of the simpler models, some harder ones can be found, with their networks and other information, in *Mathematical Models* by Cundy and Rollett.

So far the cardboard used has been from flat sheets, but "mailing tubes," containers for household cleaners, etc., provide an easy and cheap form of cylinder when required. For example it may be required to show how two equi-diameter cylinders look when they intersect at right angles with their axes in the same plane. A neat model can be made from four empty containers. Two cuts, each at 45 degrees to the axis, are made right through each cylinder (see plan view in Fig. 4).

Fig. 4.

These are preferably made by using a fine toothed hacksaw and a mitre box. The four containers can then be joined along the cut edges with balsa cement. Alternatively two containers only could be cut and stuck to a container of slightly smaller diameter. It will be interesting for the children to observe that a cut across the cylinder gives an ellipse and, for sixth-formers, that when the curved surface of the cylinder is developed, i.e., cut and opened out flat, the cut becomes a sine curve in the plane. In cutting the containers it will be found that the second cut gives a small waste portion from each. These four "scraps" can be joined together to show the space common to two cylinders intersecting at right angles. If oblique parallel lines are drawn on a rectangular sheet of thin card which is then curved and stuck to form a cylinder, a model is obtained for introducing the idea of the threads on a screw.

Circular discs are required at times. These can be obtained ready made from circular boxes, e.g., large containers of ice cream; or with a little ingenuity, a blade from one of the above mentioned knives can be fixed to a compass, and discs of any size, cut from a flat sheet. A simple model for use with latitude

and longitude can be made as follows, from two circular discs. One is used for the base, and the other is cut to give one semi-circular portion and two quadrants. Draw a diameter and a half diameter perpendicular to it on the disc forming the base. The semi-circular portion and one quadrant are then stuck along their straight edges to these lines. The model so far can be thought of as representing the equator (the circumference of the base), the northern portions of the meridian of Greenwich, 90 degrees west and 180 degrees (the curved edges of the semi-circular portion and the quadrant). The remaining quadrant is then hinged by Sellotape to the central radius of the semi-circular portion, so that its curved edge can be thought of as a variable meridian of longitude. A suitable wire ring, as used for lampshades, can be set in notches cut in the card to simulate one parallel of latitude. (Fig. V.)

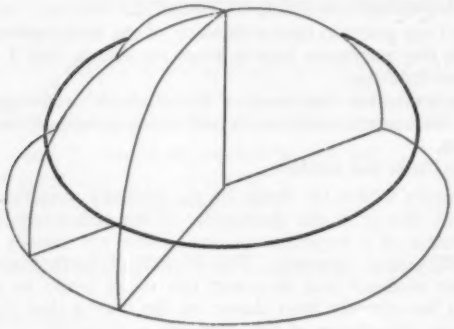


Fig. 5

If completed models are to be coloured then poster paints are best, as they give an even matt covering on cardboard. Colour should be used primarily to emphasise particular portions or points.

(To be continued)

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Advertisement in *The New Yorker*.

MATHEMATICS IN THE BBC ENGINEERING TRAINING DEPARTMENT

R. O. ROWLANDS

In the Engineering Training Department of the BBC new recruits to the Engineering Division at Technical Assistant level are given a three months' course on which they are taught the basic principles of electrical engineering, and instructed in the operation of the station equipment. On returning to their stations they are encouraged to study in their spare time to reach a standard which will enable them to benefit by a further three months' course. This course has an engineering rather than an operational bias and the problem is how to give the students a thorough understanding of the engineering aspects of the equipment without resorting to mathematics other than logarithms, simple trigonometry, complex numbers and the solution of simultaneous and quadratic equations.

In this article I am going to deal with some of the techniques adopted to enable the students to get the maximum benefit from the course and I will deal with the problem under four headings:

1. Using mathematics within the limits of the students' understanding.
2. Supplementing the mathematical result with a description of the process involved.
3. Use of analogies.
4. The use of visual aids and models.

1. *Using mathematics within the limits of the students' understanding.*

An example of this is in the derivation of the characteristic impedance and propagation coefficient of a transmission line. These are usually derived from the solution of two differential equations. This branch of mathematics is well beyond the capacity of our students and so resort has to be made to a less elegant but simpler method. It has already been shown on the course that the general formula for the characteristic impedance of any network is given by

$$Z_o = \sqrt{Z_{sc} \cdot Z_{oc}}$$

Since the characteristic impedance is independent of the length of the line we will calculate its value for a short length x of line. Such a line can be represented by the circuit shown in Figure 1 (drawn unbalanced for convenience). When x is small the series components will have a low impedance and the shunt components a high impedance.

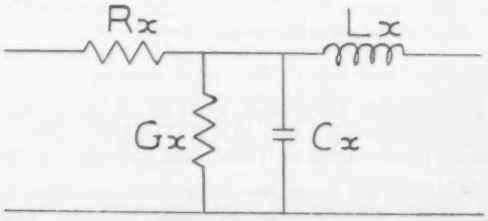


Fig. 1

Therefore $Z_{sc} \doteqdot Rx + j\omega Lx$

and $\frac{1}{Z_{oc}} \doteqdot Gx + j\omega Cx$

The approximation can be made as close as we wish by reducing x and so in the limit

$$Z_o = \sqrt{Z_{sc} \cdot Z_{oc}} = \sqrt{\frac{Rx + j\omega Lx}{Gx + j\omega Cx}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

which is therefore independent of x.

The general formula for propagation coefficient given in most text books is

$$v = \tanh^{-1}\sqrt{Z_{sc}/Z_{oc}}$$

Hyperbolic functions however do not mean much to engineers and the equivalent logarithmic expression, viz.

$$v = \frac{1}{2}\log\left\{(1 + \sqrt{Z_{sc}/Z_{oc}})/(1 - \sqrt{Z_{sc}/Z_{oc}})\right\}$$

has many advantages. It can be derived quite simply from first principles, though the derivation is too long to be given in this article.

Substituting for Z_{sc} and Z_{oc} in this expression we get that

$$v = \frac{1}{2}\log\left\{\frac{1 + \sqrt{(Rx + j\omega Lx)(Gx + j\omega Cx)}}{1 - \sqrt{(Rx + j\omega Lx)(Gx + j\omega Cx)}}\right\}$$

$$= \frac{1}{2}\log\left\{1 + x\sqrt{(R + j\omega L)(G + j\omega C)}\right\} - \frac{1}{2}\log\left\{1 - x\sqrt{(R + j\omega L)(G + j\omega C)}\right\}$$

which when x is very small

$$\doteqdot \frac{1}{2}x\sqrt{(R + j\omega L)(G + j\omega C)} - \frac{1}{2}\left\{-x\sqrt{(R + j\omega L)(G + j\omega C)}\right\}$$

$$= x\sqrt{(R + j\omega L)(G + j\omega C)}$$

2. *Supplementing the mathematical result with a description of the process involved.*

My example here will be taken from the teaching of negative feedback in amplifier circuits. It can be shown mathematically that negative feedback reduces the distortion produced by an amplifier, but negative feedback still remains rather a mysterious semi-magical process to the engineer until he is shown that the effect of subtracting a distorted output signal from the input is to predistort the input in such a way as to counteract the distortion of the amplifier. For example if an amplifier has a tendency to flatten the peak of a signal, negative feedback causes the input to be more peaky, as shown in Figure 2.

3. *Use of analogies.*

If the same mathematical process can be applied to two separate subjects teaching efficiency is increased four times. This is because the student is only required to understand one process instead of two and when the second subject is taught

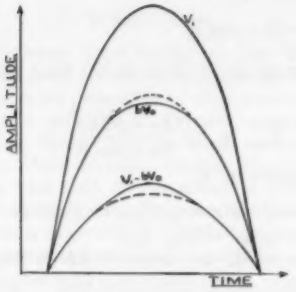


FIG 2 - PREDISTORTION OF A SIGNA
DUE TO NEGATIVE FEEDBACK.

this serves as a revision of the first subject. An example is the relation between standing waves: either electrical, mechanical or acoustical: and the envelope produced when two high frequencies are added together in a beat frequency oscillator. Standing waves are the result of two waves travelling in opposite directions. These may be represented at any instant by two vectors rotating in opposite directions the angle between them being proportional to the distance from an antinode.

The two high frequencies may also be represented by rotating vectors. If however we regard the vector representing the mean frequencies as being at rest the vector representing the lower of the two frequencies will rotate backwards relative to this while the other will rotate in a forward direction. The angle between the two vectors will be proportional to the time interval from that at which the envelope reached peak value. The same mathematical treatment may therefore be applied to both cases, bearing in mind that distance in the one case corresponds to time in the other.

4. *The use of visual aids and models.*

These enable the student to get a clear picture in his own mind of what is taking place. Again it is often possible to use the same piece of equipment to illustrate two different phenomena; and a machine was constructed in our own workshops to illustrate the relationship between (a) transverse and longitudinal wave motion, (b) amplitude and frequency modulation.

WANTED, GEOMETRY, DEAD OR ALIVE?

IAN HARRIS

Geometry is seldom discussed by our pupils outside the classroom, like other subjects, and when it is, the usual conversation centres around such questions as "What's for homework?" or "How do you do the third one?" rather than any enquiry into the structure of geometry. In the classroom there is a similar reluctance to study geometry in this manner, and there only remains a desire to juggle with geometrical facts. There is nothing creative.

These are the symptoms of "dead" geometry, which result in only the hardiest of potential mathematicians surviving to appreciate the subject. The main cause is that the development of geometry relies, too much, on "spoon-feeding": by asking leading questions, by presenting obvious diagrams and by stating properties outright. The trend is one of "Teacher goes—Class follows" and naturally quite a number of pupils are left behind. Even this is not bad enough, for the formal oral and written communication is stressed and geometry dies.

The answer to this disturbing state of affairs is to have a "living" Geometry, where interest is awakened and the trend is "Class goes—Teacher follows." A geometry in which properties are discovered, accepted or rejected by the pupils themselves, and where the geometry itself is important, the vocabulary and formal communication being learnt incidentally.

This is the challenge that must be met, and it becomes less formidable when we ask ourselves "What is Geometry?" "How is it taught?" The first question may be answered thus, "the study of invariants for a given set of transformations," or for Euclidean Geometry, our main interest in schools as "The study of the properties that remain unchanged when subjected to movement—translation and rotation." For the second question, the following are the more widely used methods of introducing geometry:

(a) From the concrete—the study of solids leading to the plane figures and their properties.
For: Knowledge is gained from experience.
Against: Gives an intuitive belief that there is only one kind of geometry.

(b) From the manipulation of geometrical instruments—the "Stage A" approach.
For: Knowledge is gained from experience.
Against: Gives an undue reliance on accurate drawings, and difficulty over the necessity for a proof.

(c) From the Axioms—a geometry from within.
For: Produces a sound logical development.
Against: Too abstract.

None of these, satisfactorily, meets the challenge to produce a self-generating geometry and there is little recognition that motion plays the vital role.

It is my belief that geometry, "live" geometry, must be built around movement, the motions of translation and rotation. With this in view, I first teach pupils to become aware, through their own experience, of these two motions and

then to apply them to the two basic notions of geometry, the straight line and the point. For example:

The translation of a point gives a point locus

The translation of a line gives parallels

The rotation of a line gives angle, etc.

Consideration of these give a real understanding of angles and parallels. Vertically opposite angles, alternate angles, etc., are soon discovered. Later, animation of more complicated configurations can give rise to such topics as the tangent, Simson Line, etc.

This is a dynamic development and is creative. I have found that the number of new facts discovered is often overwhelming and the standard of some is astonishing. Increasing evidence points to the conclusion that the ideas of geometry can readily be understood very early and that it is the transfer of this knowledge, from Teacher to Pupil as well from Pupil to Teacher, that is the hindrance.

Here is a summary of a lesson, the aim of which was to find some further properties of a parallelogram. The parallelogram had been named after the discovery, in the previous lesson, of a figure with opposite sides and angles equal, obtained by translating each of two straight lines. (Figure 1.)

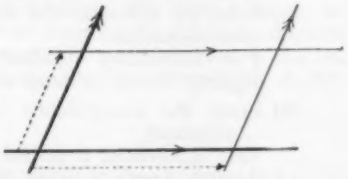


Fig. 1

Fig. 2

The lesson commenced with the production of Figure 2, an obvious extension of Figure 1. Nothing was forthcoming, except disappointment that P, Q and R were not necessarily collinear. (There could be an interesting development with areas, making P, Q and R collinear.)

Figure 3 was now suggested, being Figure 2 with AB=BC and AA′= A′A″. (Still striving for collinearity of AB′C″.) The side equalities followed. Study of this figure produced many interesting properties, e.g.

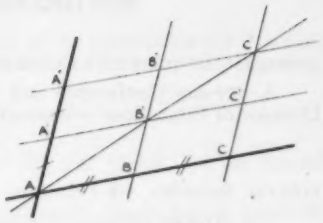


Fig. 3

1) AB′=B′C″ (AB′ translates to A′B″ which translates to B′C″).

2) AB′C″ is a straight line (AB′ has the same direction as A′B″ which has the same direction as B′C″).

3) AB′=B′C″ and AB′C″ a straight line (parm. ABB′A′ translates into parm. B′C′C″B″).

4) Diagonals of a parallelogram bisect each other (AB′=B′C″ and CB′=B′A″).

5) Mid-point theorem (in △ACC″ B′, C′ mid-points of AC″ and CC″ respectively).

The discovery of the last result was unexpected and the third was used to introduce congruence later on.

At first all this work is carried out with coloured diagrams and symbols, an oral dynamic proof being accepted. Later, with complex diagrams, the need for a letter notation becomes apparent and is invariably suggested. This leads to the formal written proof.

The film which presents a dynamic geometrical situation is an ideal method of suggesting new fields of study. (The Nicolet Films provide some fruitful topics.)

Any rigid syllabus must be disregarded for it is essential that the initiative comes from the class, but with a careful selection of discoveries, a course of work which covers the syllabus may be pursued.

This approach, I think, has gone a long way to meet our original challenge, and has its applications at all levels of geometry teaching. Pupils cease to be like robots reacting to certain geometrical stimuli, and become creative geometers with an understanding of the subject, and in time the widespread belief that Geometry is difficult compared with Arithmetic and Algebra will be diminished.

Long live "live" Geometry.

You'll forgive me if I sound a little pedantic about this. But it brings up a subject which I think should be taught along with the three R's in every elementary school. If it were so taught, gambling would be reduced from a national problem to a sporadic eccentricity, and gambling houses would close. With a very few very dangerous exceptions, the average educated citizen has no knowledge of the subject and no interest in it, and this high-mindedness costs average educated citizens of the United States many millions of dollars a year.

I'm talking about a branch of mathematics called the theory of probability. It is called by gamblers—erroneously—the law of averages.

SCARNE *On Cards.*

FORTHCOMING A.T.A.M. MEETINGS

ABERDEEN. Saturday 13th October, 1956.

A one-day Conference and Exhibition was arranged at the request of the Director of Education. A report will appear later.

LONDON. Saturday, 9th February, 1957.

The Annual General Meeting of the Association will be held at the Institute of Education, Malet Street, W.C.1, from 10-11 a.m. in the Assembly Hall. It is hoped that as many members as possible will attend.

There will be an Exhibition of Teaching Aids, open to members and friends, on the same day and in the same place, from 11 a.m. to 1 p.m. and from 2-4 p.m. The programme has been planned so that the actual making of these aids can be seen demonstrated and explained, and "workshop talks" will be given at the times stated:

11.00 a.m.	Models	J. W. Peskett
11.20 a.m.	Filmstrips	R. H. Collins
11.40 a.m.	Mechanical Apparatus	L. Swinden
12.00	Films	T. J. Fletcher and I. Harris
12.20 p.m.	Elastic Band Models ...	R. H. Fielding.

The use of the Cuisenaire rods will be explained personally to all who are interested.

The exhibition will also be open during the afternoon, and in addition Mr. T. J. Fletcher will lecture on *The Design of Mathematical Films*, in Room 151, at 2.45 p.m.

The day should be of considerable interest to teachers in Primary, Secondary Modern, and Secondary Grammar Schools, and members should bring it to the notice of their colleagues and students, all of whom will be made welcome.

ST. ANNE'S. Arrangements are being made for a weekend conference for teachers in all types of Secondary schools at St. Anne's, Lancashire, which it is hoped will be held in March, 1957. If this materialises fuller details will be circulated later. Enquiries can be addressed to Mr. C. Birtwistle, 1 Meredith Street, Nelson, Lancs.

LEICESTER. Saturday in October, 1957.

Arrangements for a one-day Conference and Exhibition are being worked out in conjunction with the Leicester Teachers' Association. Fuller details will be given in the next issue of this bulletin.

R. H. C.
R. M. F.

ANNUAL GENERAL MEETING, 1956

Minutes of the Third Annual General Meeting of the Association for Teaching Aids in Mathematics, held at Hengrove School, Bristol, on Saturday, 4th February, 1956, at 10 a.m.

Present: the Chairman, the Secretaries and twelve members.

Apologies from R. D. Knight and R. H. Fielding.

Minutes. It was proposed, seconded and carried that the minutes of the Second Annual General Meeting be taken as read.

Amendment to the Constitution. The following amendment was proposed, seconded and carried: "The initial subscription of any new member joining after 1st October shall also cover the subsequent calendar year." This wording to be inserted in paragraph four, after "payable to the Treasurer from the beginning of the year."

The Secretary was instructed to see that members joining in the middle of the year should receive copies of the previous Bulletin if these were available.

Report of the Secretary (Mr. R. H. Collins). In giving a brief survey of the activities of the Association over the past twelve months, he expressed the opinion that the success of the increased number of one-day Conferences organised in different parts of the country augured well for the future. He looked forward to a rapid growth in membership during 1956-7 and thought that the printed Bulletin would help to confirm the contacts made at conferences. It was to be hoped that some of the new members would play an active part by giving demonstration lessons, writing articles for the Bulletin and drawing up scripts for films and filmstrips. There was an urgent need for someone to assume responsibility for the collection of Infant and Junior School apparatus so that the Exhibitions and Conferences should cover all age ranges adequately.

The groundwork for the first film to be produced by the Association's film unit had been prepared by Mr. Fletcher and Mr. Harris.

Thanks were due to the many who had worked hard for the Association during the year; in particular to the Chairman, Dr. Gattegno; to the Treasurer, Miss Briggs; to the Joint Secretary, Miss Giuseppi; to Mr. Birtwistle for his work in the Manchester area, and to all members of the Committee.

Treasurer's Report. Subject to audit, the balance at the end of the financial year was £94 11s. 7d., and 184 members had paid subscriptions.

Mr. J. W. Peskett and Mr. C. Hope were appointed as auditors.

The reports were adopted.

Election of Committee Members and Officers.

(a) Chairman: Dr. C. Gattegno.
(b) Joint Secretaries: R. H. Collins, Miss Y. Giuseppi.
(c) Treasurer: Miss B. Briggs .
(d) Committee: C. Birtwistle, T. J. Fletcher, R. H. Fielding, Mrs. Fyfe, I. Harris, C. Hope, J. W. Peskett and J. V. Trivett.

Any Other Business. Because of the small attendance it was agreed that the minutes should be printed in Bulletin No. 3 and comments on decisions invited from other members.

R. H. COLLINS (Joint Secretary).

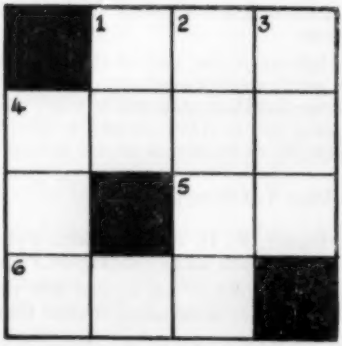
BALANCE SHEET OF THE A.T.A.M.

February 16th, 1955 — December 31st, 1955

INCOME	£	s.	d.	EXPENDITURE	£	s.	d.
By balance, February, 1955	23	0	4	To Treasurer's postage ...		7	2
Subscriptions	34	8	6	,, ,, stationery		5	9½
Sale of films	30	0	0	Purchase of films	20	0	0
Proceeds from Lyme Hall weekend	7	18	2	Assistant secretary, postage on bulletins, stationery, etc.	6	0	0
Mathematical Pie, special account	50	0	0	Cost of duplicating bulletin	19	1	0
				Cost of half return fare for member to attend London committee meeting	1	11	5
				Expenses for Birmingham weekend	3	10	0
				Balance carried forward to 1956	94	11	7½
	£145	7	0		£145	7	0

B. BRIGGS, (Honorary Treasurer).

CROSS FIGURE



CLUES ACROSS :	CLUES DOWN :
1. $y + 2z$	1. y
4. $x^2\ (x-1)$	2. x^3
5. $3x$	3. z
6. x^2	4. $x^2 + z$

Complete the cross-figure finding x, y and z.

Solution on page 29.

IAN HARRIS.

FILM DAY

On Saturday, May 28th, some sixty people attended a programme of Mathematical films suitable for use in teaching at the College of Preceptors, London. Almost from the start the audience had to bear the brunt of frustration with stoppages, breaks, burnt film and loss of focus from the projector and projectionist hired by the Association for the occasion. Fortunately, however, this did not spoil the day. Another projectionist was found and by 4 p.m. those present had witnessed a host of worthwhile films and film-strips.

The meeting commenced with a demonstration lesson led by Dr. Gattegno before a class of girls from Putney High School. They were shown part of a film and after circles had rolled upon other circles on the screen for a few minutes the children were asked what they had seen. We kept silent and watched from the back. Gradually, the pupils revealed varied experiences. Question followed upon question 'a la Gattegno': "Is this right?", "Tell me what you saw," "Can you put that into a sentence?", "Will you draw it for us, please?" Precise answers meant moving on, vague statements called for marking time until light percolated from the majority.

When the girls had returned to their well-deserved Saturday break, the audience had the opportunity to put Dr. Gattegno on the spot: "What was your aim?", "What did you achieve?", "If muddle occurs with a bright class like this what would have happened to a C stream?" Dr. Gattegno smiled and ping-ponged back most remarks. These questions, asked at most of our conferences, were not the ones to consider; what mattered most was that we had witnessed a group of children reacting to a film on mathematics in the light of their own experiences. The girls had said what they had said; it was our task to interpret, to learn from them and especially to ponder upon their difficulties as revealed by their muddled and eager-to-please remarks. A little later, after some more films, Dr. Gattegno summarised the lesson for us, surprised us by telling us that the class was, in fact, C stream and one visitor, at least, was sent to lunch regenerated in the belief that we had learnt once again from those we teach and were better equipped, therefore, for our return to the classroom.

Mr. T. Fletcher introduced all the films, including his own production, *Plucked Strings*. We also saw some of the Nicolet films, his latest—shown in public for the first time ever—*The Common Generation of the Three Conics*, startlingly and beautifully emphasising just what its title says. These, with *Regular Polygons*—another fascinating film to watch—*The Cardioid* and others were films of line drawings; *Friction* and the American *Slide Rule* employed ordinary pictures with animated drawings as well as spoken commentaries.

The film-strips were not screened but Mr. I. Harris displayed a selection for us to see. Time was taken during the afternoon for this and for twenty minutes we had the chance of extending our arms to the light, viewing sample frames from each strip and deciding which would be suitable for our own circumstances.

Finally, Mr. Fletcher announced that the Association, through the generosity of the Nuffield Foundation, had acquired a camera and would, in future, be able to make its own films. Not only that: if mathematical films were to be made in the near future they would have to be made "by the people in this room or those whom they represent." Anyone interested in any branch of the craft as pertaining to films on Mathematics was invited to convey willingness to help to the Committee.

JOHN TRIVETT.

OVERSEAS REVIEW

We are now receiving back numbers of overseas periodicals and it is proposed in each issue of *Mathematics Teaching* to give brief notes on what appear to be interesting articles in these magazines. It is hoped then to make them available to members through the Association's Library Service.

In the *Australian Mathematics Teacher*, April, 1956, is an article on the introduction of some portion of the History of Mathematics to the first three years of the High School Syllabus. The forms concerned are aged 11 to 14, and their I.Q.s range from 80 to 120. The method used is to select suitable topics, e.g., the calendar, map making, probability, intimately connected with everyday life. About 12 project sheets are prepared and these form the basis for class discussion and are designed for the mathematics display board.

Each topic is considered historically, the subjects for the project sheets starting in prehistoric days with man's inability to understand the laws of nature and therefore his inability to use them, and showing how gradually by mathematical effort his problems were slowly solved. The work is suitable for either Secondary Modern or Grammar Schools.

In *The Mathematics Teacher* (American), December, 1955, is an article which attempts to bring the concepts of mathematical logic within the grasp of pupils studying formal geometry (especially three dimensional) and, at the same time, develop a better foundation for indirect proof. After a number of introductory exercises designed to demonstrate the logical equivalence of various propositions at the same time bringing out the difficulty of such an exercise *a priori*, the writer goes on to introduce some elementary work on the propositional calculus with the usual logical constants. Work on truth tables is demonstrated. When complex propositions are translated into symbols logical equivalence is immediately obvious by consideration of their truth tables. The three types logically equivalent to "if p then q" are shown, and there the article ends.

I have experimented with this work on a class of boys in the first year of advanced level work. We had a fascinating time and went far beyond this article; but I am still hoping to find that the work on logic has helped them in their solid geometry. The writer gives no indication of how this knowledge and experience is applied to geometry!

In *Mathematics & Pædagogia*, No. 8, a terminal review published by the Belgian Society of Mathematics Teachers, there is an interesting article entitled *The General Gas Laws and their Geometric Properties*. This short article, which barely permits of a review, gives an elegant proof, by using the geometry of ruled surfaces (p, v, T being the co-ordinates used) that if a gas obeys two of the laws it obeys the third. The article is in French and would interest the Scholarship mathematician or physicist.

R. D. KNIGHT.

FOREIGN PERIODICALS SERVICE

The magazines mentioned in the above article may be borrowed by members from R. D. Knight, King's School, Worcester.

Fourpence to cover postage should be sent on application, and the magazine should be returned within a fortnight. Applications will be dealt with in strict succession but there may be some delay if the demand is great.

A NEW FILMSTRIP PRODUCTION

As the result of a recent article in *Mathematics Teaching* a start has been made on the production of a filmstrip on *The Characteristics of Sine and Cosine Graphs and the Construction of Fourier Series.*

So far the first draft script has been drawn up by two members of the Association, and they would welcome any advice and help which other members may be able to give. Constructive suggestions at this stage would ensure that the strip will be of use not only at Sixth Form level and above, but also lower levels.

Anyone interested in viewing the script should write to the Secretary of the Association.

The Audio-Visual Center, Indiana University, Bloomington, Indiana, U.S.A., distributes a series of films under the general title *Understanding Numbers.* The titles of the separate films are *The Earliest Numbers, Base and Place, Big Numbers, Fundamental Operations, Short Cuts, Fractions,* and *New Numbers.* Each film is in black and white and lasts thirty minutes, and the series is described in an illustrated pamphlet, *Understanding Numbers: Their History and Use,* obtainable at $1.00 from Ulrich's Bookstore, 547 East University Avenue, Ann Arbor, Michigan, U.S.A.

The geometrical films by J. L. Nicolet, which have been seen at many A.T.A.M. demonstrations are now distributed in the United Kingdom by The Cuisenaire Company Ltd., 14 Basingstoke Road, Reading.

Owing to the possibility of confusion with another address the collection of mathematical filmstrips previously known as "Gateway Filmstrips" is now to be called "Mathematical Pie Filmstrips." Copies may be purchased, or obtained for review from 97 Chequer Road, Doncaster, Yorks.

OUR CONTRIBUTORS

Readers may have seen reports in the national Press about the work of the BBC Engineering Training Department at Wood Norton, in Worcestershire, and the article by R. O. ROWLANDS describes the mathematics teaching in fuller detail. Our Association has done very little investigation of the difficult problem of presenting mathematical ideas to technologists who are working on inadequate foundations, and we hope that this article will draw attention to this important matter.

J. W. PESKETT is regularly in charge of the exhibitions of models arranged by the Association, and his article is the first of a series which will describe the construction of many of the models which have been displayed. The cost of professionally made models is very often prohibitive, and these articles will assist those who wish to make their own.

This number contains an Overseas Review by R. D. KNIGHT, of King's School, Worcester. We hope this will become a regular feature, and he will be pleased to receive any overseas journals covering a similar field to our own.

We would thank the Technical Drawing department of the Secondary Technical School, Nelson, for preparing some of the illustrations for this bulletin.

BOOK REVIEWS

"Didactique de l'Initiation Mathématique à l'Ecole Primaire." Doctoral thesis by B. Hardi Fischer, Geneva, 1955. B.I.E. Price 6 frs.

In this thesis Mr. Fischer undertakes the justification of an active approach to the beginnings of number work on the grounds of Piaget's psychology.

The work is divided into two parts, the first concerned mainly with a description and assessment of methods, the second with the psychological theory on which the author's views are based.

The reader will welcome the summary of much Continental and American literature on methods that are well established in many schools, and the assessment of their relative values. We find, for example, a chapter on the verbal method and drill, which the author considers as being things of the past, although in fact they still prevail, unfortunately, in many junior schools. Another chapter is concerned with what are described as intuitive and pictorial methods. In this section there are detailed accounts of the work done by Lay, Kuehnel and Montessori, whose contributions are later criticised for their failure to take account of action in the learning process. The use made of pictures and intuition in more recent years, by American authors, in particular Carleton Washburn, and the German writers F. Drenckhahn and J. Whittman, is then dealt with. Fischer quotes as being more particularly intuitive the methods developed by Nicolet with his films, by Emma Castelnuovo for the beginnings of geometry and by Catherine Stern for number work. The third section relates Dewey's ideas to modern methods and to Piaget's psychology. Dewey is given the credit he deserves for having seen clearly that action by the learner is essential in the formation of mathematical ideas. His views on number are less impressive than are those of Decroly and of Audemars and Lafendel before 1930, and since then, those of de May, Spitzer, Walper and Cuisenaire, all of whom are however deeply indebted to Dewey.

The psychological part of the work is obviously more biased, since the author is a disciple of Piaget and is here writing a thesis under his direction. It contains many interesting comments which make it worth studying, but in the reviewer's opinion the restatement of a system elaborated with genetic epistemological intentions is not necessarily adequate to what is involved in the process of re-invention by children of mathematics. Experiments with children constantly show that Piaget's conception of number and of fraction is too remote from theirs for teachers to base their methods on it. This weakness in Mr. Fischer's approach causes him to miss altogether the opportunity he might have taken of understanding, for example, what Cuisenaire's method means for the primary school. Since it is assumed that only Piaget's ideas are adequate to a modern way of teaching arithmetic, the whole study suffers from a bias which to those with different experience seems somewhat naïve. It should not be too much to ask that the writer of a doctoral thesis who subjects all earlier proposals to a critical examination should examine as keenly his own views. It is somewhat disappointing to find that a work published in 1955 proves to be in so many ways a work of the past.

C. GATTEGNO.

Know Your Maths. Part I, by A. Keith and A. Martindale, published by Blackie & Son. Price 7s. 3d. with answers, 6s. 9d. without answers.

The type and setting out is attractive and this book competently achieves what it sets out to do, that is, to give plenty of examples through which the rules of arithmetic can be used and applied.

In particular, the problems and revision exercises contain examples that will be meaningful for the pupil, and 36 of the 149 pages of the text are well used in this way. The exercises on units: length, speed, weight, capacity and time, show imagination, as does a set of examples on a petrol pump. The scale drawings and graph chapters are good and so is the work on the points of the compass.

The authors admit the limitations they have set to their approach, in the preface, by the words: "The attempt . . . has been made . . . to present the various topics in such a way that the pupil will understand *in some measure* what he is asked to do." (The italics are mine.) No attempt has been made to interest the pupil in inverse operations, or in factors, when the four rules are revised. It seems as if care has been taken to avoid introducing any mathematical thought into the chapters on decimals and fractions.

The introduction of "maths" into the title is hardly justified as algebraic and geometrical topics are presented solely to give practice in arithmetic. In fact, the authors seem to have no faith that mathematics has any hope of being meaningful to the pupils, as mathematics.

The work on geometry is almost entirely of a numerical nature but clear within these set limits. One criticism should be made; on page 110 it is stated that "A quadrilateral with its opposite sides parallel is called a parallelogram." I should feel justified, on that definition, in giving the trapezium (b) as an answer to the question. The exercise on circles is sadly lacking in imagination.

Similar remarks apply to algebra, which is introduced by substitution, and operations are firmly ignored in favour of rules to be memorised. This is the policy of despair of the authors, who state, on page 129; "Meanwhile you must *try* to become so accustomed to using letters that you can handle them as easily as numbers." (Again, the italics are mine.)

I recommend this book for drill practice in arithmetic, but not for the beginnings of a course in mathematics.

R. M. FYFE.

From Simple Numbers to the Calculus. By Egmont Colerus, translated by B. C. and H. F. Brookes. Pp. xi, 255. 12s. 6d. (Heinemann.)

Colerus's avowed intention is to bring a simple explanation of mathematics to the layman. Written in a popular style, well set out, the result is a very readable, interesting book.

Starting from a consideration of scales of notation, the author is soon explaining permutations and combinations. Algebra includes the early introduction of a limit and the problem of 0/0. Negative numbers arise from brackets and are illustrated by "borrowing and owing," a disappointing treatment, but it is unusual to find multiplication explained:

$$(-a) \times (+b) = -(+ab) = -ab.$$

Indices and polynomials, multiplied and divided, follow. Fractions are thought worthy of a chapter to themselves and their treatment includes a consideration of $\lim a/x$ as x tends to infinity. The section on equations includes Euler's method of solving a simple indeterminate equation. These elementary considerations conclude with a simple treatment of irrational numbers in which series for π and e are mentioned.

The second part of the book is concerned with the development of functional ideas in which a well judged mixture of topics on trigonometry, co-ordinate geometry, complex numbers, quadratics, A.P. and G.P. and differentiation and integration leads to a novelty of presentation. Some topics are merely introduced: trigonometrical ratios are defined by the revolving radius of a quadrant of a circle, but are only used to measure the gradient of a line.

For a student wishing to bridge the gap between the ordinary and the advanced level or who wishes to draw together the ideas of the alternative syllabus, this book would prove extremely useful. It contains no examples for the reader to work; it is a book to read, leaving topics at a point where the student would like to read further. The addition of a bibliography to carry the student further would have made the book more suited to its purpose. The book is well worth a place in the school library.

C. HOPE.

Mathematical Test Papers for Upper Forms. By R. Blamire Clarke. University of London Press Ltd. Paper, 2s.

Since the introduction of the G.C.E. ordinary level alternative syllabus, examiners and teachers have responded with varying degrees of imagination to its possibilities, but most writers of text-books have only been inspired to the extent of producing books with successive chapters of arithmetic, algebra and geometry—the mixtures as before but now in one volume!

The present booklet is a collection of examples only, so that its contribution to the treatment of the syllabus is negligible. However, it does give a useful collection of eighteen test-papers of the type usually set on the alternative syllabus. Most of the papers are intended to last two hours and contain a Section A of seven questions and a Section B of six. Calculus has been included in the later papers, and other topics include most of those found in the syllabuses of the various examining boards (a notable exception is the problem involving scale-drawing from a three-dimensional figure as set by Cambridge).

This book should be of use where volumes of past papers are not published or where a teacher requires additional revision test material.

C. BIRTWISTLE.

The Theory of Numbers, by Burton W. Jones. Constable and Co. Pp. 143. 24s.

This book was written for an American college course and covers the following ground—the number system, repeating decimals, congruences, Diophantine equa-

tions, continued fractions and quadratic residues. It is intended as a first introduction, and throughout carefully selected questions encourage the student to experiment and to make discoveries for himself. The author often uses a chain of simple examples to lead up to a theorem, and he seeks to give evidence of the reasonableness of postulates before formulating them. One wishes that more books adopted this style. The methods of proof employed are simple and elegant and there is a large number of well-chosen examples; but their value is reduced since no solutions are given. Where the interest of a topic warrants it, results are stated without proof and the reader is given a reference to more advanced works in which proofs may be found.

In *Mathematics and Plausible Reasoning* Polya recently advocated number theory as a mathematical discipline which can be used to give instruction in many types of logical thinking, inductive as well as deductive. Professor Jones provides a useful classbook for any teacher who wishes to experiment extensively on these lines.

The book is excellent for the purpose for which it was written, but few teachers in England will want to cover anything like the same ground at the same level of instruction. The author states that the first chapter, on the development of the number system, requires about twenty class periods. The earlier sections of this chapter provide a thorough treatment of the natural numbers and the elementary laws of algebra, and they are to be commended for their forward-looking point of view. The discussion focuses attention on just those elementary properties of numbers which are of most importance in the later development of abstract algebra. American books are often good in this respect, and our own are often rather weak. But the treatment of real numbers (by means of decimal sequences) and complex numbers at the end of this chapter is far too brief to be of any value in English undergraduate instruction.

The rest of the book deals with aspects of number theory which are usually taken as an advanced subject in English universities, when students are prepared to take a rather tougher book than this one. However anyone wishing for a book on number theory which goes as far as such topics as Wilson's theorem, Pell's equation and the Law of Quadratic Reciprocity and which could be used with First Year university students or with the Third Year Sixth will find it very useful.

T.J.F.

BOOKS RECEIVED FOR REVIEW

Introductory Comprehensive Mathematics—J. D. Hodson (MacMillan).
Know Your Mathematics—A. Keith and A. Martindale (Blackie).
Mathematical Test Papers for Upper Forms—R. Blamire Clarke (Uni. Lond. Press).
Know Your Mathematics (Part II)—A. Keith and A. Martindale (Blackie).

FILM REVIEW

Common Generation of the Three Conics. Animated Geometry, No. 22, by J. L. Nicolet. Drawing and animation by Lucien Motard. Black and white. Silent. 16 mm. 4 mins.

Nicolet's style in mathematical film making is by now well-known and his latest production is in the manner which we have come to associate with him. That is to say he takes a simple geometrical situation involving movement and develops it without the use of words or any conventional lettering. This film is a re-make of some of the scenes from *Théorèmes et Problèmes*, which has been shown in this country but not put into general circulation, but it is different from all of his previous films in that the animation has been done by Motard, the artist who drew the three mathematical films made for the French Ministry of Education under Cantagrel's direction.

In this film we see the generation of conic sections as the locus of the centre of a circle which touches a fixed circle and passes through a fixed point. There are four closely related scenes in which the locus is in turn a circle, ellipse, parabola and hyperbola. The generation of the hyperbola is a most happy climax because the two branches of the curve are generated by a continuous construction and the asymptotes of the curve have an interesting relation to the tangents from the fixed point to the fixed circle. This film makes a good problem situation for pupils to investigate since the approach to the loci is one which they will not expect and further properties of the conics can be derived from it. Nicolet is a master of understatement, and this film, like so many of his others, suggests many more geometrical properties than the few which it portrays directly. Some of these suggestions are so interesting that one feels regret that one or two, at least, have not been pursued further.

The continuity employed between scenes two and three, and three and four raises the question "what is the family of conics generated in this way?" The answer is a system with one focus and one extremity of the major axis fixed. This is a degenerate case of a two-point two-line system. How grand it would have been to see the entire system animated as a pattern of the kind that Motard has drawn in the earlier films with which he has been associated!

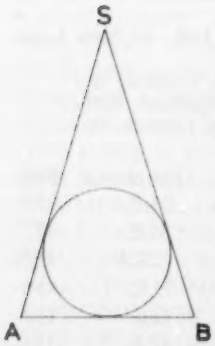
Motard always produces animation of the very first quality, and this present film is no exception. The format of the drawings is attractive and the movement smooth. The choice of a suitable tempo is always the animator's most difficult problem, and here the difficulty is triumphantly overcome. The whole action flows with effortless grace, and it is a delight to see Nicolet's ideas presented with photography of fully professional quality.

In spite of the obvious gains when this film is compared with the earlier ones on the same theme one cannot fail to notice with regret that there has also been a certain loss. The earlier films incorporate animated proofs of the results which are (in the reviewer's opinion) probably the best examples of rigorous logic which have been seen in mathematical films so far. They completely refute the suggestion so often heard that geometrical films are "very nice but they prove nothing." In these

earlier films the construction given was shown to be equivalent to another previously known construction; and this proved that the locus was truly the ellipse or hyperbola that it appeared to be. It is possible that Nicolet decided to omit the proofs here because although the ellipse, parabola and hyperbola are generated by a common construction, and whilst the three cases are continuously deformable into one another the three proofs do not behave in quite the same way, and each case calls for significantly different treatment. It would not be easy therefore to incorporate them artistically into what is, as it stands, an exceedingly artistic film. But it would have been interesting to see an attempt made to overcome the difficulty. Mathematical films are not only delightful designs they are also logical structures and the pursuit of logical rigour is one of the most urgent problems confronting us in the development of the art; and Nicolet with his thorough appreciation of the power of the wordless, unlettered diagram and his practical teaching experience has perhaps more to tell us than any other designer.

This is without doubt the way to make a film about geometry. So see it for yourself, show it to the Sixth Form, show it to students, and show it to your colleagues who are not yet convinced.

T. J. F.



SNOOKERED. (Solution to the problem in *Mathematics Teaching* No. 2.)

Correspondents tackled this puzzle by various methods, but no solution was as neat as the one proposed by Mr. E. Hogben who set the problem. A "penthouse" of two intersecting planes can be dropped over any ball and its ridge oriented to contain the light source, S. The diagram shows the situation viewed along the line of intersection of these two planes. Each shadow is then contained between the traces of the sides of the "penthouse" on the table, and every minor axis is equal to AB. Furthermore each ball stands on a focus of its shadow, but this is not required in the problem.

SOLUTION TO CROSS-FIGURE PUZZLE ON PAGE 20

$x = 21 \qquad y = 88 \qquad z = 403$

MATHEMATICAL FILMSTRIPS AND FILMS

FILMSTRIPS

GREAT BRITAIN

All filmstrips are black and white unless otherwise stated.

Key (24) Number of frames.

S Single frame size.
D Double frame size.
N With notes.
* Have been presented to the Association for exhibition and research purposes by their Producer or Distributor. Detailed reports of these will be published.

Distributors

D.K. Dance-Kaufmann Ltd., 11-13 Victoria Street, Liverpool 2.
E.F.V.A. Educational Foundation for Visual Aids, 33 Queen Anne Street, London, W.1.
E.P. Educational Productions Ltd., East Ardsley, Wakefield, Yorks.
E.S.A. Educational Supply Association, 181 High Holborn, London, W.C.1.
G.B. Gaumont-British Film Library, Aintree Road, Perivale, Greenford, Middlesex.
M.P. Mathematical Pie Filmstrips, 97 Chequer Road, Doncaster, Yorks.
P. Sir Isaac Pitman and Sons, Ltd., Pitman House, Parker Street, Kingsway, London, W.C.2.
P.P. Picture Post Education Department, Hulton Press Ltd., 43 Shoe Lane, London, E.C.4.
U.H. Unicorn Head Visual Aids Ltd., 35 Portland Place, London, W.1.
V.I.S. Visual Information Service, 12 Bridge Street, Hungerford, Berks.
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TIME AND MOTION STUDY	E.S.A.	16/6

UNITED STATES OF AMERICA

The following list has been compiled from various American journals. It is not known if any have yet found their way into this country, and any further information about them would be greatly appreciated.

A study of measurement

1. LINEAR MEASURE. (40)
2. STORY OF MEASUREMENT. (32)
3. LINES, ANGLES AND SURFACE MEASUREMENT. (36)
4. AREAS OF RECTANGLES, PARALLELOGRAMS. (37)
5. SPECIAL MEASUREMENT, BOARD TEST, RIGHT TRIANGLE. (43)

Photo and Sound Productions, 116 Natoma Street, San Francisco, 5, California.

Fraction Series

1. MEANING OF FRACTIONS. (45)
2. CHANGING THE TERMS OF FRACTIONS. (46)
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7. MULTIPLYING FRACTIONS AND MIXED NUMBERS. (62)
8. DIVIDING FRACTIONS AND MIXED NUMBERS. (47)

Stanley Bowman Company, 513 West 166th Street, New York City.

Man and Measuring Series (Colour)

1. EARLY COUNTING
2. EARLY MEASURING
3. EARLY TIME TELLING
4. GEOMETRIC FIGURES

The Filmstrip House, 15 West 46th Street, New York, 36.

THINKING IN SYMBOLS. (27)
GROUPING SYMBOLS AND ORDER OF OPERATIONS. (34)
GEOMETRIC FIGURES. (29)
MEASUREMENT. (36)
VARIABLES AND CO-ORDINATES. (33)
MATHEMATICS IN DAILY LIVING. (28)

McGraw Hill Book Company, 446 West 42nd Street, New York City.

FRACTIONS, DECIMALS AND PERCENTAGE. (50)
ORDER OF OPERATIONS. (46)

Jam Handy Organisation, 2821, East Grand Boulevard, Detroit, 11, Michegan.

LITERACY IN MATHEMATICS. (33) (Colour)

Bureau of Audio-Visual Instruction, University of Nebraska, Lincoln, Nebraska.

KNOWING NUMBERS. (all colour)
FRACTION FACTS
FRACTION FORMS
SOME SUMS
DIFFERENCE DIFFERENCES
PLEASING PRODUCTS
QUOTIENT QUESTS
FRACTION FINDING
ZERO — THE FORGOTTEN NUMBER

Eyre Gate House Inc., 2716 Forty-first Avenue, Long Island City, 1, N.Y.

HOW LIFE INSURANCE BEGAN. (44)
HOW LIFE INSURANCE OPERATES. (41)
HOW LIFE INSURANCE POLICIES WORK. (42)
PLANNING FAMILY LIFE INSURANCE. (46)

Institute of Life Insurance, 488 Madison Avenue, New York, 22.

FILMS

Unless otherwise stated all these films are in black and white.

FAMILIES OF LINES, FAMILIES OF PARABOLAS. 16 mm. Silent. 20 mins.
SIMPLE REGULAR POLYGONS. 16 mm. Silent. 20 mins.
SIMPLE GEOMETRICAL LOCI. 16 mm. Sound (English or French). 15 mins.

These three films are produced by Cantagrel for the French Ministry of Education; the script is by Jacquemart and the animation by Motard. They may be hired from the Institut Francais du Royaume Uni, Queensberry Place, S.W.7.

The films were intended for revision use in French schools for pupils of ages 15 and upwards. The scope of the films is indicated by the titles. Detailed synopses may be obtained from I. Harris, Esq., 122 North Road, Dartford, Kent.

GENERATION OF INVOLUTE GEAR TEETH. 16 mm. or 35 mm. Silent. 4 mins.

HYPOCYCLIC MOTION. 16 mm. or 35 mm. Silent. 9 mins.

$x'' + x = 0$. 16 mm. or 35 mm. Silent. 3 min.

$x'' + x = A \sin nt$. 16 mm. or 35 mm. Silent. 8 mins.

Designed by R. A. Fairthorne and animated by Brian Salt. Copies may be purchased from R. A. Fairthorne, Esq., Kirk Michael, Clockhouse Road, Farnborough, Hants., or hired from the British Film Institute, 164 Shaftesbury Avenue, London, W.C.2.

The films on differential equations are animations which may be regarded as working models of a differential analyser. The presentation is highly original, and teachers have never realised full merits of these films because the subject is treated

in such an unconventional manner. The film on gear teeth is easier to follow, and the film on hypocyclic motion should be shown to all Sixth Form geometers and can profitably be used in conjunction with the films on hypocycloids and epicycloids listed below.

THE SIMSON LINE. 16 mm. Silent. 13 mins.
THE CARDIOID. 16 mm. Silent. 15 mins.

Produced by Sir John Cass College, Jewry Street, London E.C.3. Directed and animated by T. J. Fletcher. Copies on hire from the college, and on sale from the college or from the A.T.A.M. (R. H. Collins, 97 Chequer Road, Doncaster).

These films cover advanced topics in Sixth Form geometry. *The Simson Line* contains a good deal on the three-cusped hypocycloid (which is the envelope of the Simson line) and *The Cardioid* illustrates theorems which are true of epicyclic curves in general.

TRANSFER OF POWER. 16 mm. or 35 mm. Sound. About 20 mins.

Made by the Shell Film Unit and available free from the Petroleum Film Bureau.

A "popular" documentary film on the history of the toothed wheel which illustrates many mechanical principles and contains animated sequences on the constructions of gear teeth and on the law of the lever. A very useful film for beginners in mechanics, combining instruction and entertainment. (The film is quite free of any objectionable advertising matter.)

THE CALCULATOR THAT REMEMBERS. 16 mm. Sound. Colour.

An advertising film made for the Burroughs Calculating Machine Company, distributed by the Gaumont British Film Library, free if with a programme of other films.

This film does not illustrate any mathematical principles, but it does enable a class to see a business calculating machine in action.

THE SLIDE RULE: MULTIPLICATION AND DIVISION.
16 mm. Sound. 24 mins.
THE SLIDE RULE: PERCENTAGES, PROPORTION AND SQUARE ROOTS.
16 mm. Sound. 21 mins.

Made for the U.S. Office of Education and distributed in Great Britain by the Central Film Library, Government Buildings, Bromyard Avenue, Acton.

These were made as training films for the armed forces and are excellent films of their kind. There is no explanation of the theory of the slide rule, the emphasis is rather on achieving results by performing a certain drill; but animation techniques are used to convey instruction on reading scales which it would be difficult to do in any other way.

The following "Animated Geometry" films by J. L. Nicolet are available in this country.

I—Lower and middle school.

1. A circle passing through three points.
2. Locus of centre of circles tangent to two concentric circles.

3. Locus of centre of circles tangent to a given circle and passing through the centre.
4. Circle as locus of vertex of a fixed angle between two lines.
5. Segments containing an angle.
6. Internal bisectors.
7. External bisectors.
8. Angles at the circumference.
9. Locus of contact points of parallel tangents to equal circles passing through a given point.
10. Construction of the regular pentagon.
11. The golden section and the regular pentagon.
12. Triangles formed of sides of regular polygons.
13. Locus of points from which two fixed circles are seen under equal angles.

II—Upper school (to scholarship levels).

14. Hypocyclic motion with circles in the ratio 1 to 2.
15. The golden section and the strophoid.
16. Poles and polars in the circle.
17. Ellipse as locus of a point on a line of fixed length sliding upon two perpendicular lines.
18. Ellipse as locus of centres of circles tangent to a fixed circle and passing through a fixed point (interior).
19. Hyperbola as locus of centres of circles tangent to a fixed circle and passing through a fixed point (exterior).
20. Parabola as locus of centres of circles tangent to a fixed line and passing through a fixed point.
21. Parabola as locus of centres of circles orthogonal to a given circle and tangent to a fixed line.
22. Common generation of the three conics.

All of these films are 16 mm. and silent. They can be obtained from The Cuisenaire Co., Ltd., 14 Basingstoke Road, Reading, agent for the Nicolet films in the U.K.

The filmstrip section of this list has been compiled by Mr. I. Harris, Dartford Grammar School, Dartford, Kent; and the film section by Mr. T. J. Fletcher, 11 Oak Grove Road, S.E.20. We wish to keep the list up to date, and to reissue it from time to time, and we will be pleased to receive additional information or corrections. This work will be further assisted if the Association is named when enquiries are made to distributors.

T.J.F.
I.H.

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Contributions to the journal should be sent to the Executive Editor. Contributors are asked to keep mathematical notation as typographically simple as they can, and to submit illustrations, where necessary, in a form suitable for reproduction.